

AN OUTLINE FOR THE ANALYSIS OF WAVE INDUCED BOTTOM PRESSURE FLUCTUATIONS

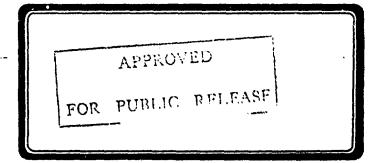
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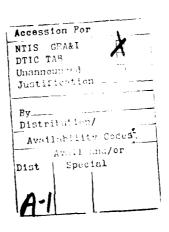
AN OUTLINE FOR THE ANALYSIS OF WAVE INDUCED BOTTOM PRESSURE FLUCTUATIONS

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ABSTRACT

This note outlines a computer program to replace the manual Draper-Tucker analysis of bottom pressure fluctuations induced by surface waves.





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AN OUTLINE FOR THE ANALYSIS OF WAVE INDUCED BOTTOM PRESSURE FLUCTUATIONS

1. INTRODUCTION

This note outlines a computer based method, developed by the author at MRI. (Sydney), for the analysis of wave induced bottom pressure fluctuations.

As a part of the Mine Warfare Pilot Survey (MWPS), bottom pressure fluctuations were measured in the Australian theatre between 1972 and 1984 using uifferential type transducers on loan from the US Navy. The outputs from these transducers were recorded in analogue form both on paper chart and on magnetic tape as a frequency modulated signal. Routinely, the paper chart records were analysed manually by the Draper-Tucker method [1] and yielded only the median amplitude and the zero crossing period. Only a small number of the paper chart records have been analysed manually in greater detail to determine wave amplitude and period distributions. A few select magnetic tape records have been digitised and analysed along the lines outlined in this note.

Since 1986 bottom pressure fluctuations have been measured using a system developed around a Digiquartz pressure transducer, manufactured by Paroscientific Inc. Pressure related frequency output from the transducer (in the range 40 kHz to 36 kHz) is measured by a frequency counter and converted to pressure and recorded digitally at a sampling rate of 2 Hz on a NEC APC III computer.

Since bottom pressure fluctuations are, in fact, attenuated surface waves [2], the method outlined in this note is based on the techniques and terminology used for the analysis of surface waves. Thus pressure fluctuations are referred to as wave heights/amplitudes and generally expressed in mm of water and not in Pa.

Coastal and ocean wave statistics are often classified into three categories, the short term statistics, the medium term wave climate statistics and the long term statistics.

The short term statistics are concerned with the distributions of individual wave heights and periods in a given sea state, such as:

- a) Distribution of individual wave heights.
- b) Distribution of individual wave periods.
- c) Joint distribution of wave heights and periods.
- d) Distribution of individual wave steepness.
- e) Statistics of wave groups.

The medium term wave climate statistics are concerned with the distributions of sea state parameters such as the significant wave height, zero crossing periods and others in the time frame of month, season and year. Distributions of sea state parameters include:

- a) Marginal distributions of wave height, period and direction parameters.
- b) Joint distributions of wave height, period and direction parameters.
- c) Statistics of persistency of sea state.

The long term wave statistics deal with distributions of individual waves or sea state parameters and corresponding estimates of extreme values relating to return periods of the order of 50 or 100 years.

The analysis described in this note is basically concerned with the analysis of individual bottom pressure records, i.e. with the short term statistics. Many parameters are used to characterise the behaviour of water surface displacement in the gravity wave frequency band. The International Association for Hydraulic Research (IAHR) lists over 100 such parameters. The current program computes only a fraction of all the possible parameters and not even all of them are required for routine analysis of bottom pressure records, and hence an option exists to run an abbreviated analysis. Likewise, since the program has been developed in-house, new statistics that may become of interest can be readily incorporated. Among such statistics, which are studied by coastal and ocean wave researchers and which could be of interest to pressure mine warfare, are the correlation coefficient between the heights and periods of individual waves and the occurrence and properties of wave groups.

Short term statistics collected over a number of years enable the deduction of the medium term wave climate statistics, which are essential for the planning of mine warfare operations. They are used for the estimation of the effects that the waves might have on the effectiveness of pressure mines during a particular month, season or year. The long term statistics are of limited interest to mine warfare studies.

Bottom pressure fluctuations are analysed in the time, the frequency and the probability domains. The basic analysis is performed in the time and the frequency domains. The required input for both analyses is the time series of a wave record and the results of the two analyses are largely complementary. The probability domain analysis is required in those instances when only the pressure fluctuation frequency spectra is available, as, for instance, is the case when the bottom pressure fluctuations are deduced from the surface wave spectra. In the routine analysis of bottom pressure fluctuation time records, the probability domain analysis provides a comparison between the actual and predicted wave parameters.

Examples of the analysis of two bottom pressure records are shown in Appendix A.

2. OUTLINE OF ANALYSIS

The general outline of the program is illustrated in Figure 1. In each of the domains there are a number of distinct computational modules. Although, as the flow chart indicates, some exchange of information is necessary among the domains, for the purposes of discussion of the program the domains will be considered separately.

The length of a wave record must be a compromise between a long record to achieve adequate statistical stability and wave stationarity. A record length of 17 min 4 s (2048 data points at 2 Hz sampling rate) is an often used compromise length in surface wave analysis and hence has been also selected as the compromise length for bottom pressure fluctuation analysis. Longer records, when available, are best subdivided into separate records of 2048 data points, unless wave stationarity can be established.

2.1 Time Domain

2.1.1 Module A - High-pass Filtering

The Digiquartz system measures the absolute pressure on the sea bottom, which is the sum of the static head (water depth plus constant barometric pressure) and the fluctuations due to wave action and barometric pressure changes.

For routine analysis of bottom pressure fluctuations only those fluctuations that are within the bandwidth of ship pressure signatures, i.e. between approximately 0.003 Hz and 0.3 Hz are of interest. The low pass-filter is provided by the medium itself, since wave attenuation with depth increases rapidly with increasing frequency. A second order Butterworth high-pass filter, with a cut-off frequency of 0.001 Hz, is used to suppress pressure changes due to very low frequency waves, such as tides. The actual filtering is performed in the frequency domain. Prior to high-pass filtering, the static head effects are removed by subtracting from the entire record the mean pressure determined from the first 200 data points. After filtering, in order to minimise any transient effects, the first 300 data points are discarded.

No attempt has been made to allow for the measurement and compensation of the pressure changes due to barometric pressure variations. To do this a second Digiquartz transducer based system would be required to provide an independent measurement of the barometric pressure and it is only in exceptional circumstances that the barometric pressure changes would be of such rapidity and magnitude that the high-pass filter would be unable to deal with them.

2.1.2 Module B - Analysis of Wave Parameters

The program performs a wave-by-wave analysis of the filtered time record. Individual waves are defined by successive zero upcrossings, with the zero crossing points determined by linear interpolation. Wave heights are determined by the differences between the highest crest and lowest trough, the crests and troughs being points where the water surface has a local maximum or minimum respectively. Wave amplitude is defined as one half of the corresponding wave height. The program also computes:

- a) the number of zero upcrossings, Nz
- b) the number of waves, N
- the total number of crests, N_C c)

The individual wave heights are analysed to obtain the following height parameters:

- a)
- h)
- the highest wave, H_{max} the mean of the highest one-tenth waves, $H_{1/10}$ the mean of the highest one-third waves or the significant wave height, $H_{\rm S}$ c)
- the root mean square wave height, H_{rms}
- e)
- the mean wave height, H_m the median wave height, H f)
- the median wave neight, n_{med} the mean wave amplitude (defined as one-half of the mean wave height), A_{m} g)

Individual wave periods are determined by the differences between successive zero upcrossings. They are analysed to determine:

- the upper quartile, Qu a)
- the lower quartile, Q b)
- the interquartile range, IQR c)

The program also determines:

- a) the zero crossing period, Tzt
- b) the crest period, Tct.

These periods are obtained by dividing the total length of time of a record by the number of zero upcrossings and the number of crests in the record, respectively.

In the time domain Tzt and Tct enable the estimation of the spectral width parameter, $\epsilon_{t}[3,4]$:

$$\epsilon_{\mathbf{t}}^2 = 1 - (T_{\mathbf{ct}}/T_{\mathbf{zt}})^2 \tag{1}$$

The spectral width parameter is a measure of the rms width of the wave energy density spectrum; its value ranges from 0 to 1. If ϵ_{+} is small, the spectrum is narrow banded, if ϵ_{+} is near 1, the spectrum is broad banded. Spectra are considered reasonably narrow banded if ϵ_{t} is below 0.60.

The program also computes the variance of the time series, V.

The above parameters constitute the abbreviated time domain analysis and are listed in Tables 1A and 1B of Appendix A under the subheading Time Domain for Wave Records A and B, respectively. Wave Record A is an example of a reasonably narrow band record ($\epsilon_t=0.519$), while Wave Record B ($\epsilon_t=0.745$) is an example of a broader band record.

In the full analysis the program also computes and lists the individual wave amplitudes and periods (Tables 2A and 2B), the wave heights in decreasing order of magnitude (Tables 3A and 3B), the crest heights in decreasing order of magnitude

(Tables 4A and 4B) and the wave periods in decreasing order of magnitude (Tables 5A and 5B).

2.1.3 Module C - Amplitude and Period Distribution

In the full analysis the program also determines the distribution of the normalised amplitudes and periods. Following Longuet-Higgins [5], the actual values are divided by the rms amplitude $(2\ m_o)^{1/2}$ and the average period (m_o/m_1) , respectively. The spectral moments m_o and m_1 are obtained from the frequency domain analysis (see Section 2.2.2). The number of waves occurring within bandwidths of 0.2 of the normalised amplitudes and periods are determined and expressed as fractions of the total number of waves within each band and also as the corresponding densities per unit normalised amplitude and period respectively. They are illustrated in Tables 6A and 6B, 7A and 7B in Appendix A.

The joint distribution of wave amplitudes and periods is obtained by determining the number of waves occurring within a grid determined by the normalised amplitude and period bands and are expressed as percentages of the total number of waves (Tables 8A and 8B). Following Shum and Melville [6], a smoothed joint distribution is also computed by modifying each grid point value as weighted average of its original value and its eight neighbouring points, i.e.

$$X(i,j) = x(i,j)/4 + (x(i-1,j) + x(i+1,j) + x(i,j-1) + x(i,j+1))/8$$

$$+ (x(i-1,j-1) + x(i+1,j-1) + x(i+1,j+1) + x(i-1,j+1))/16$$
 (2)

in which x(i,j) is the original value at the grid point (i,j) and X(i,j) is the smoothed value (Tables 9A and 9B).

2.2 Frequency Domain

2.2.1 Module D - Spectral Density

A standard FFT algorithm is used to compute the spectral densities, S(f), with S(f) being defined for positive values only of the frequency, f. The record length of 17 min 4 s results in a frequency resolution of 1/1024 Hz, but with only 2 degrees of freedom. The program provides for the statistical stability to be improved by subdividing the record into smaller segments and by averaging the spectral densities. However, the resulting frequency resolution could be too coarse for the resolution of waves in the infragravity regime (0.003 to 0.03 Hz). Alternatively, the program allows for the statistical stability to be improved by averaging over a number of adjacent spectral densities. A number of data windows, such as the rectangular, Hamming and Welch, can be selected.

In the examples cited in Appendix A the entire record (2048 data points) has been taken as the FFT length. A rectangular window has been applied and the spectral densities have been averaged over 7 adjacent values, resulting in 14 degrees of freedom. Although the use of the rectangular window increases the leakage, it has the advantage that it ensures that the wave power in the frequency domain (the zeroth moment, m_0 , see Section 2.2.2) equals the power in the time domain (variance, V), which is essential for the probability domain estimates of wave parameters and

possible subsequent numerical simulation of time records. The spectral densities for Wave Records A and B are shown in Figures 2 and 3. It will be noted that Wave Record B has a well defined peak in the infragravity regime.

2.2.2 Module E - Spectral Moments

The spectral density of the wave record, S(f) is used to determine the following spectral moments m_n :

$$m_0 = \int_0^\infty f^0 S(f) df$$
 (3)

$$m_1 = \int_{-\infty}^{\infty} f^1 S(f) df$$
 (4)

$$\mathsf{m}_{(-1)} = \int_0^\infty \mathbf{f}^{-1} \mathsf{S}(\mathbf{f}) \ \mathsf{d}\mathbf{f} \tag{5}$$

$$m_2 = \int_0^\infty f^2 S(f) df$$
 (6)

$$m_4 = \int_0^\infty f^4 S(f) df$$
 (7)

Frequency domain calculations suffer from the disadvantage that the spectral density contains a high frequency tail, which, theoretically, is proportional to the minus fifth power of the frequency. This can lead to some disturbing effects on some commonly used wave parameters and is sometimes denoted "parameter instability". Values of the spectral moments (stability) depend on the choice of the low-and the high-frequency cut-offs. Any uncertainty in the spectral densities at the higher frequencies is magnified in the calculations of the higher moments, e.g. m., because of the fourth power of the frequency involved in its estimate. For this reason the computations of spectral moments have been terminated at 0.5 Hz.

2.2.3 Module F - Spectral Parameters

Two spectral width parameters, $\epsilon_{\mathbf{f}}$ and ν are computed from the spectral moments.

It has been shown by Cartwright and Longuet-Higgins (3) that an equivalent expression to the time domain spect all width parameter, $\epsilon_{\rm t}$, can also be derived in the frequency domain in terms of the spectral moments, viz.

$$\epsilon_{\mathbf{f}} = (1 - m_2^2/m_0 m_4^2)^{1/2}$$
 (8)

Since ϵ_f depends on the fourth spectral moment m_a , and thus depends rather critically on the behaviour of the spectrum at the higher frequencies, Longuet-Higgins [5], has proposed another spectral width parameter ν , which depends only on the three lowest moments m_0 , m_1 and m_2 :

$$v = (m_0 m_2 / m_1 - 1)^{1/2}$$
 (9)

This parameter, which also tends to zero for narrow band spectra, has been used by Longuet-Higgins in his derivation of a theoretical distribution for periods and amplitudes of random waves (Section 2.3).

Since ϵ_f is sensitive to errors in the estimates of m₂ and, particularly, m₄, Goda [7] has derived a spectral peakedness parameter defined as

$$Q_{\rm p} = 2/m_0^2 \int_0^\infty f S^2(f) df$$
 (10)

which depends only on the first moment of the power density spectrum. There is no direct relationship between ϵ_f and Q_p . However, a small ϵ_f implies that Q_p is large and vice versa. In the present context the interest in Q_p is connected with the empirical observation that the tendency of wave group formation increases with increasing value of Q_p .

2.2.4 Module G - Spectral Periods

The corresponding expressions to the time domain zero crossing and the crest periods may also be derived in the frequency domain in terms of the spectral moments [3], viz.

$$T_{zf} = (m_0/m_2)^{1/2}$$
 (11)

$$T_{cf} = (m_2/m_u)^{1/2}$$
 (12)

Another period, which can be expressed in terms of the spectral moments, is the average period, $T_{\rm av}$. This period is used in some spectral formulations, such as the International Ship Structure: Congress (ISSC) spectrum and is given by:

$$T_{av} = m_o / m_i$$
 (13)

For use in wave power calculations the appropriate period is the average energy period, $T_{\rm e}$. This period, which strictly speaking should be called the average power period, is given by:

$$T_{\mathbf{e}} = m_{(-1)}/m_0 \tag{14}$$

Another often used period is the peak period, $\boldsymbol{T}_{\boldsymbol{p}}$ i.e. the period at which $S(\boldsymbol{f})$ is maximum.

The values of all of the above periods, with the exception of the peak period, depend to some extent on the choice of the high-frequency cut-off.

The spectral mon.ents, spectral parameters and the frequency domain estimates of wave periods are all listed in Tables 1A and 1B under the subheading Frequency Domain.

2.3 Probability Domain

2.3.1 Module H - Wave Heights via Rayleigh Distribution

A number of wave height distributions have been proposed. The analysis described in this note considers only the most widely used distribution, namely that due to Longuet-Higgins [8], who has shown that if the sea surface is assumed to be the sum of many sine waves in random phase, and if the frequency spectrum is sufficiently narrow, then the wave heights are distributed according to a Rayleigh distribution, viz

$$p(H) = 2H/H_{rms}^{2} \exp(-(H/H_{rms})^{2})$$
 (15)

where p(H) is the probability density function, H is the wave height and H_{rms} is the rms wave height. The sea surface which satisfies the above requirements is considered to be Gaussian. The Gaussian sea surface assumes symmetry about the still water level and has a zero mean.

The Rayleigh distribution enables the estimation of the probability domain characteristic wave heights corresponding to those determined in the time domain analysis (Section 2.1.2). Shown below are the expressions for the estimates of the relevant wave heights. All, except for the highest wave, can be expressed in terms of only the zeroth spectral moment, m_0 (or, alternatively, in the terms of the rms wave height, H_{rms}). They are all subject to the statistical variability caused by the use of finite record lengths. The expected or the mean maximum wave height, H_{rms} , depends fundamentally on the length of the wave record and hence its estimation requires the rumber of waves, N, as an additional input from the time domain analysis.

$$\overline{H}_{max} = H_{rms} (\{\ln(N)\}^{1/2} + 0.28861|\ln(N)\}^{-1/2})$$
 (16)

$$H_{1/10} = 1.800 \ H_{rms} = 5.091 \ (m_0)^{1/2}$$
 (17)

$$H_{S} = 1.416 H_{TMS} = 4.005 (m_0)^{1/2}$$
 (18)

$$H_{\text{rms}} = 1.0 \ H_{\text{rms}} = 2 \ (2 \ \text{m}_{\text{o}})^{1/2}$$
 (19)

$$H_{m} = (\pi)^{1/2}/2 H_{rms} = (2\pi m_{0})^{1/2}$$
 (20)

$$H_{\text{mud}} = (\ln 2)^{1/2} H_{\text{rms}} = (8 \ln 2 m_0)^{1/2}$$
 (21)

$$A_{\rm m} = H_{\rm m}/2 = (\pi)^{1/2}/4 H_{\rm rms} = (2\pi m_0)^{1/2}/2$$
 (22)

The probability domain estimates for the above wave heights are listed in Tables 1A and 1B under the subheading Probability Domain.

The Rayleigh distribution has proven to be a reliable measure of the wave height distribution for waves in deep water. It also has been used to describe wave heights in finite depth water with reasonable success if the assumption of a Gaussian sea surface is not violated to a great extent. However, as waves begin to break in shallow water, they become nonsinusoidal in shape, the Gaussian sea surface assumption is violated and appreciable deviations from the Rayleigh distribution can occur. The standard predictive expression for wave breaking is $H_{\rm b}=0.78{\rm d}$, where $H_{\rm b}$ is the breaking wave height and d is the depth. It is, however, suggested in [9] that the above expression underestimates the breaking wave height and that a representative breaking wave height should be taken as equal to the depth. It follows that only in exceptionally high sea states can breaking waves be expected to occur in depths of interest to pressure mine warfare.

For waves with finite band-width or those exhibiting some nonlinearities the Rayleigh distribution somewhat overestimates the characteristic wave heights. Longuet-Higgins [10] has shown that in such cases the distribution of wave heights can still be described by the Rayleigh distribution, provided the rms wave height, $H_{\rm rms}$ is estimated from the original data and not from the frequency spectrum. Alternatively, good agreement for storm waves with the Rayleigh distribution has been obtained [10] if the rms amplitude is computed from

$$A_{rms} = 0.925 (2m_0)^{1/2}$$
 (23)

instead of Equation (21).

Since the medium acts as a low-pass filter the pressure fluctuations on the bottom are of a narrower band-width than the correpsonding surface waves, which should improve the validity of the Rayleigh distribution in all water depths of interest to mine warfare. In the analysis the probability domain wave heights are determined from the zeroth spectral moment.

${\bf 2.3.2}\quad Module~I~~Amplitude~and~Period~Distribution$

A wave train is only partially described by its height distribution. A more complete description requires also the knowledge of the wave period distribution and the joint distribution of wave heights and periods. Although a number of such theoretical distributions have been proposed, the program currently considers only the distributions proposed by Longuet-Higgins [5].

In [5] Longuet-Higgins proposes theoretical distributions for the wave amplitude density, the wave period density and the joint density of wave amplitudes and periods based on narrow-band theory and which depend only on the lower-order spectral moments m_0 , m_1 and m_2 . As a measure of the spectral width Longuet-Higgins uses the parameter ν , defined by him in Equation (9) in Section 2.2.3. It is suggested that to be consistent with the narrow-band assumption, ν should be less than

0.6. Then warrow-band assumption also implies that the spectra must be unimodal.

The probability density of the normalised wave amplitude is given by

$$p(R) = 2 R e^{-R^2} L(v) F(R/v)$$
 (24)

where R is the amplitude normalised with respect to the rms amplitude, $L(\nu)$ is the normalisation factor which accounts for the fact that only positive values of the normalised period T are considered and is given by

$$1/L = 0.5 \left[1 + (1+v^2)^{-1/2}\right]$$
 (25)

and

$$F(R/\nu) = 1/\pi^{1/2} \int_{-\infty}^{R/\nu} \exp(-\beta^{2}) d\beta$$
 (26)

is the error function. Equation (23) states that the density of R almost follows a Rayleigh distribution but must be corrected by the factor $L(\nu)$ $F(R/\nu)$. For values of R that are of the order of 1 or larger the correction is small. However, when R is small, the correction becomes significant.

In the program p(R) is evaluated for 40 values of R in steps of 0.1 R.

The probability density of the normalised wave period is

$$p(T) = (L/2\nu T^2) \left[1 + (1-1/T)^2/\nu^2\right]^{-3/2}$$
(27)

This density is asymmetric and, unlike p(R), is very sensitive to ν .

In the program, p(T) is evaluated for 31 values of T in steps of 0.1 T.

The probability densities of amplitudes and periods are illustrated in Tables 10A and 10B of Appendix A.

The joint density of wave amplitudes and periods is given by:

$$p(R,T) = (1/\pi^{1/2}\nu)(R^{2}/T^{2}) \exp(-R^{2}[1+(1-1/T)^{2}/\nu^{2}])L(\nu)$$
 (28)

For comparison with the actual joint distribution of the wave amplitudes and periods, computed in Module D of the time domain analysis, the theoretical joint density values are also smoothed and expressed as percentages of waves on the same grid (Tables 11A and 11B).

Longuet-Higgins [5] reports that the above theoretical joint distribution agrees quite well with certain observed wave data. Ambiguities that exist in the comparisons of the theoretical distributions with those obtained from field data often arise from the (often quite arbitrary) definitions of wave height and period used in data analysis.

3. CONCLUSIONS

A computer program has been developed for the analysis of short term statistics of bottom pressure fluctuations induced by surface waves. The program has been developed for the analysis of digitised records and is based on the techniques and terminology used for the analysis of surface waves. It replaces the Draper-Tucker manual method, which was used for the analysis of analogue chart records. Bottom pressure fluctuations are analysed in the time, frequency and probability domains. The in-house developed program meets all present requirements for the routine analysis of bottom pressure fluctuations and can be readily extended to include new statistics which may become of interest.

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APPENDIX A

This Appendix illustrates the results of the full analysis for two bottom pressure records. Record A is representative of a wave record having a relatively narrow energy density spectrum ($\nu=0.253$), while Record B has a broad energy density spectrum ($\nu=0.759$).

Unless normalised, all wave heights/amplitudes are expressed in millimetres, periods in seconds and frequencies in hertz. The spectral densities for the Records A and B are illustrated in Figures 2 and 3, respectively.

WAVE ANALYSIS - WAVE RECORD A

File Name: Wave Record A - Length: 2048 Date: 18 November 1986 - Station: B FFT Length: 2048 Rectangular Window

Table 1A

	Time Domain	Probability Domain
Number of zero upcrossings, N _z	113	
Number of waves, N	112	
Total number of crests, N _c	131	
Time series variance, V	2417.0	
Highest wave, Hmor	300.7	321.5
Highest wave, H _{max} Mean of 10% highest waves, H _{1/10}	228.1	250.3
Significant wave height, H	190.9	196.6
rms wave height. H	135.9	139.0
Mean wave height, Hm	121.9	123.2
Median wave height, Hmad	118.5	115.8
Mean wave height, H _m Median wave height, H _{med} Mean wave amplitude, A _m	60.9	61.6
		Frequency
		Domain
Spectral width, e	.519	.444
Spectral width, v		.253
Peakedness parameter, Q _p		3.15
Spectral moment: mo		2416.4
m_1		265.55
m ₂		31.046
$m_{_{4}}$.4967
m ₍₋₁₎		28106.6
Zero crossing period, Tz	9.06	8.82
Crest period, T _c	7.74	7.91
Average wave period, T _{av} Average energy period, T _e		9.10
Average energy period, Te		11.63
Peak period, Tp		8.83
Period lower quartile, Q1	7.61	
Period upper quartile, $Q'_{\mathbf{u}}$	10.40	
Interquartile range, IQR	2.79	

Table	2A	Wave amplitu					
112	7.6	79	8.7	59	8.0	79	9.9
101	9.4	72	9.1	89	9.5	105	10.3
92	7.8	50	10.5	93	9.7	34	18.1
70	10.2	32	11.9	74	9.4	70	8.4
23	4.9	96	8.2	87	8.4	42	7.6
75	16.2	113	9.8	41	6.1	41	12.5
57	8.9	48	8.4	60	13.4	65	9.4
69	9.8	50	8.6	29	7.9	86	10.3
53	9.8	35	7.4	104	9.0	112	7.8
56	7.3	59	7.9	52	10.4	34	11.4
30	10.0	29	11.3	50	14.7	73	11.1
22	6.2	49	11.9	60	7.1	18	4.4
37	6.5	64	7.5	53	18.6	112	9.9
66	8.0	90	8.6	63	7.2	34	5.4
40	6.7	72	12.0	95	11.2	36	11.6
60	6.8	103	10.5	79	7.4	74	9.4
35	8.2	14	9.4	103	11.2	123	8.6
57	8.2	40	10.5	43	9.2	15	7.1
87	8.8	93	8.0	53	8.5	32	10.8
12	4.1	. 3	4.3	13	8.3	20	10.5
33	7.0	103	9.3	35	8.4	7	5.5
26	7.3	116	10.6	86	9.9	79	10.7
61	8.1		11.2	59	8.5	24	6.5
19	11.6	40	8.6	29	6.6	17	7.9
46	7.3	3 78	9.1	64	8.7	35	6.9
88	9.1		8.0	74	10.8	11	4.9
87	10.5		14.8	56	7.4	97	8.0
87	8.8	57	6.3	150	8.2	85	8.4

Table	3A	Wave heights	in decreas	ing order of	f magnitude	e	
300	246	233	226	225	224	224	210
208	207	206	206	202	195	193	190
187	186	185	181	178	177	175	175
175	175	173	173	170	169	159	159
158	158	156	150	149	148	148	147
144	144	141	140	139	132	130	129
128	127	122	121	121	120	119	119
118	115	115	114	113	112	106	106
106	104	101	100	100	100	98	96
92	87	85	83	83	81	80	80
75	74	72	71	70	70	70	68
68	68	67	65	64	60	59	58
58	53	48	47	45	41	38	36
34	31	29	26	24	22	15	6

142	134	130	128	128	122	120	119
113	113	110	110	103	103	103	99
97	94	93	93	93	89	88	87
86	85	85	85	84	84	83	82
79	79	79	78	78	77	76	75
75	73	72	71	71	70	68	67
66	65	64	62	62	61	60	60
57	57	56	54	54	53	53	53
50	49	49	48	48	47	46	46
46	44	44	44	43	42	40	36
35	34	33	30	29	29	29	28
27	26	26	26	26	25	25	25
23	23	22	22	21	19	18	18
18	17	16	16	14	14	14	14
12	12	11	11	7	7	6	5
5	3	2	-2	-4	-7	-7	-7
-8	-10	-21					

Table 5A	Wave	e periods i	n decreasi	ng order of magnitude							
18.55	18.05	16.20	14.77	14.69	13.40	12.51	12.01				
11.92	11.89	11.63	11.59	11.38	11.28	11,23	11.16				
11.16	11.09	10.83	10.78	10.74	10.64	10.53	10.51				
10.48	10.47	10.46	10.40	10.32	10.30	10.24	10.04				
9.89	9.88	9.87	9.83	9.80	9.79	9.74	9.48				
9,44	9.44	9.43	9.41	9.35	9.27	9.19	9.10				
9.10	9.05	8.98	8.90	8.83	8.80	8.72	8.70				
8.63	8.62	8.61	8.59	8.52	8.51	8.42	8.40				
8.37	8.37	8.37	8.30	8.23	8.22	8.19	8.15				
8.14	8.03	8.01	8.01	7.95	7.95	7.94	7.88				
7.86	7.80	7.75	7.61	7.58	7.47	7.41	7.37				
7.36	7.34	7.28	7.26	7.22	7.13	7.05	7.03				
6.92	6.80	6.66	6.64	6.54	6.52	6.30	6.24				
6.15	5.50	5.39	4.94	4.85	4.41	4.33	4.11				

Table 6A Normalised amplitudes and amplitude density

Norm. Ampl.	Fract. Waves	Density
.02	.045	.223
.24	.089	.446
.46	.196	.982
.68	.116	.580
.8 - 1.0	.161	.804
1.0 - 1.2	.125	.625
1.2 - 1.4	.152	.759
1.4 - 1.6	.054	.268
1.6 - 1.8	.054	.268
1.8 - 2.0	.000	.000
2.0 - 2.2	.009	.045
2.2 - 2.4	.000	.000
2.4 - 2.6	.000	.000
2.6 - 2.8	.000	.000
2.8 - 3.0	.000	.000
3.0 - 3.2	.000	.000
3.2 - 3.4	.000	.000
3.4 - 3.6	.000	.000
3.6 - 3.8	.000	.000
3.8 - 4.0	.000	.000

Table 7A Normalised periods and period density

Norm. Per.	Fract. Waves	Density
.02	.000	.000
.24	.000	.000
.46	.054	.268
.68	.143	.714
.8 - 1.0	.366	1.830
1.0 - 1.2	.277	1.384
1.2 - 1.4	.107	.536
1.4 - 1.6	.009	.045
1.6 - 1.8	.027	.134
1.8 - 2.0	.009	.045
2.0 - 2.2	.009	.045
2.2 - 2.4	.000	.000
2.4 - 2.6	.000	.000
2.6 - 2.8	.000	.000
2.8 - 3.0	.000	.000

Table 8A Joint period and amplitude scatter diagram

Relati	ve An	plitu	de												
3.9	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.(
3.7	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.(
3.5	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	
3.3	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	
3.1	.0	.0.	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	
2.9	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	
2.7	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	
2.5	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	
2.3	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	
2.1	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	
1.9	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	
1.7	.0	.0	.0	.0	2.7	2.7	.0	.0	.0	n.	.õ	.0	.0	.0	
1.5	.0	.0	.0	.0	.9	3.6	.9	.0	.0	.0	.0	.0	.0	.0	
1.3	.0	.0	.0	.0	8.9	5.4	.9	.0	.0	.0	.0	.0	.0	.0	
1.1	.0	.0	.0	.0	3.6	6.3	1.8	.0	.9	.0	.0	.0	.0	.0	
.9	.0	.0	.0	4.5	8.9	1.8	.0	.9	.0	.0	.0	.0	.0	.0	
.7	.0	.0	.0	.9	3.6	3.6	1.8	.0	.9	.0	.9	.0	.0	.0	
.5	.0	.0	.9	5.4	4.5	2.7	4.5	.0	.9	.9	.0	.0	.0	.0	
.3	.0	.0	1.8	2.7	1.8	1.8	.9	.0	.0	.0	.0	0,	.0	0.	
.1	.0	.0	2.7	.9	.9	.9	.0	.0	.0	.0	.0	.0	.0	.0	
	.1	.3	.5	.7	.9	1.1	1.3	1.5	1.7	1.9	2.1	2.3	2.5	2.7	2

Table 9A Smoothed joint period and amplitude scatter diagram

Relati	ve Am	plitu	de								_				
3.9	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
3.7	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
3.5	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
3.3	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
3.1	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
2.9	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
2.7	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0).
2.5	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	
2.3	.0	.0	.0	.1	.1	.1	.0	.0	.0	.0	.0	.0	.0	.0	
2.1	.0	.0	.0	.1	.2	.1	.0	.0	.0	.0	.0	.0	.0	.0	
1.9	.0	.0	.0	.2	.6	.6	.2	.0	.0	.0	.0	.0	.0	.0	.0
1.7	.0	.0	.0	.4	1.3	1.6	.7	.1	.0	.0	.0	.0	.0	.0	
1.5	.0	.0	.0	.8	2.6	2.9	1.3	.2	.0	.0	.0	.0	.0	.0	
1.3	.0	.0	.0	1.4	4.1	4.2	1.8	.3	.1	.1	.0	.0	.0	.0	
1.1	.0	.0	.3	2.1	4.6	4.3	1.8	.5	.3	.1	.0	.0	.0	.0	
.9	.0	.0	.6	2.8	4.6	3.5	1.4	.6	.3	.2	.1	.1	.0	.0	
.7	.0	.1	.8	2.8	4.0	3.2	1.8	.8	.4	.4	.3	.1	.0	.0	
.5	.0	.2	1.3	2.9	3.3	3.0	2.1	.9	.4	.4	.2	.1	.0	.6	
.3	.0	.4	1.6	2.5	2.2	1.7	1.2	.4	.2	.2	.1	.0	.0	.0	
.1	.0	.4	1.2	1.2	.8	.5	.2	.1	.0	.0	.0	.0	.0	.0	
	.1	.3	.5	.7	.9	1.1	1.3	1.5	1.7	1.9	2.1	2.3	2.5	2.7	2.

Table 10A Probability domain

Amp	litude and Period De	nsities
R&T	p(R)	p(T)
.0	.0000	.0000
.1	.1367	.0044
.2	.3290	.0126
.3	.5244	.0279
.4	.6810	.0577
.5	.7882	.1184
.6	.8497	.2488
.7	.8709	.5378
.8	.8567	1.1284
.9	.8132	1.9021
1.0	.7472	2.0077
1.1	.6662	1.3828
1.2	.5774	.8117
1.3	.4872	.4788
1.4	.4005	.2983
1.5	.3211	.1970
1.6	.2512	.1371
1.7	.1919	.0996
1.8	.1432	.0750
1.9	.1044	.0581
2.0	.0744	.0462
2.1	.0518	.0374
2.2	.0353	.0309
2.3	.0236	.0259
2.4	.0154	.0219
2.5	.0098	.0188
2.6	.0061	.0163
2.7	.0037	.0143
2.8	.0022	.0126
2.9	.0013	.0111
3.0	.0008	.0100
3.1	.0004	.0089
3.2	.0002	.008
3.3	.0001	.0073
3.4	.0001	.006′
3.5	.0000	.006:
3.6	.0000	.0056
3.7	.0000	.005
3.8	.0000	.0048
3.9	.0000	.004

Table 11A Joint distribution of smoothed wave periods and amplitudes, p(R,T)

Relati	ve Am	plitu	de												
3.9	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
3.7	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	,C
3.5	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
3.3	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
3.1	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
2.9	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	. (
2.7	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	0,	
2.5	.0	.0	.0	.0	.1	.1	.0	.0	.0	.0	.0	.0	.0	.0	
2.3	.0	.0	.0	.0	.2	.2	.0	.0	.0	.0	.0	.0	.0.	.0	
2.1	.0	.0	.0	.0	.5	.5	.0	.0	.0	.0	.0	.0.	.0,	,()	
1.9	.0	.0	.0	.0	1.0	1.0	.1	.0	.0	.0	.0	0,	.0.	0,	
1.7	.0	.0	.0	.1	1.8	1.7	.2	.0	0.	0.	0.	0.	0.	θ ,	
1.5	.0	.0	.0	.2	3.0	2.7	.5	.1	.0	.0	.0	.0	.0	0,	
1.3	.0	.0	.0	.5	4.3	3.7	1.0	.2	.0	.0	.0.	.0	0,	n,	
1.1	.0	.0	.0	1.0	5.6	4.6	1.6	.4	.1	.0	.0	.0	.0	.0	
.9	.0	.0	.0	1.8	6.2	4.8	2.1	.8	.3	. 1	.1	.0	.0	.0.	
.7	.0	.0	.2	2.8	5.7	4.2	2.2	1.0	.5	.3	.2	. 1	. 1	.0.	
.5	.0	.0	.6	3.2	4.0	2.8	1.7	1.0	.6	.4	.3	.2	.1	. 1	
.3	.0	.2	1.3	2.2	1.9	1.3	.8	.6	.4	.3	.2	.2	. 1	. 1	
.1	.0	.5	.7	.5	.3	.2	.2	.1	.1	.1	.1	0,	.0	0,	
	.1	.3	.5	.7	.9	1.1	1.3	1.5	1.7	1.9	2.1	2.3	2.5	2.7	2.

WAVE ANALYSIS - WAVE RECORD B

File Name: Wave Record B - Length: 2048 Date: 20 May 1987 - Station: E FFT Length: 2048 Rectangular Window

Table 1B

	Time Domain	Probability Domain
Nur per of zero upcrossings, Nz	69	
Number of waves, N	68	
Total number of crests, No.	102	
Time series variance, V	747.6	
Highest wave, Hmay	124.6	170.1
Highest wave, H _{max} Mean of 10% highest waves, H1/10	110.2	139.1
Significant wave height, H _s	83.6	109.3
rms wave height, H _{rms}	67.9	77.3
Mean wave height, H _{med}	63.8	68.5
Median wave height, H _{med}	61.0	64.3
Mean wave amplitude, A _m	°1.9	34.3
		Frequenc
		Domain
Spec, al width, e	.745	.742
Spectral width, v		.759
Peakedness parameter, Q _p		2.27
Spectral moment: m ₃		746.6
m_1		41.22
m ₂		3.586
m ₄		.0383
m ₍₋₁₎		61067.4
Zero crossing period, T _z	14.96	14.43
Crest period, T _C	9.98	9.68
Average wave period, Tav		18.11
Average energy period, Te		81.80
Peak period, Tp		12.64
Period lower quartile, Q1	10.96	
Period upper quartile, Q ₁₁	13.48	
Interquartile range, IQR	2.51	

Table 2B Wave amplitudes and periods

43	12.5	55	12.1	50	11.3	38	12,1	
44	11.1	37	10.0	26	31.3	30	7.8	
25	10.4	20	10.3	17	14.0	28	13.7	
38	11.7	41	12.6	49	59.9	20	12.0	
31	12.7	30	11.1	24	15.0	14	10.8	
28	11.4	38	37.0	41	13.0	39	13.2	
29	12.0	38	12.2	26	15.6	14	10.6	
18	12.7	20	8.9	34	12.0	36	11.6	
30	13.3	58	12.1	48	12.9	24	11.6	
23	12.8	36	13.7	34	40.0	23	12.2	
2 2	11.0	23	15.9	13	13.1	17	12.5	
13	6.3	23	10.9	56	75.4	5	5.5	
7	7.8	31	14.2	24	15.9	28	11.0	
33	10.4	29	10.7	30	10.8	34	26.6	
46	13.2	24	13.1	32	10.3	35	13.3	
30	14.2	38	12.4	33	23.2	41	11.4	
30	13.5	33	12.0	46	10.8	62	10.9	

Table 3B Wave heights in decreasing order of magnitude

-								
	124	117	112	110	100	98	97	93
	92	89	87	83	82	82	79	77
	76	76	76	76	74	72	72	70
	69	69	69	66	66	66	64	62
	62	61	61	61	60	60	60	59
	58	57	57	56	53	52	50	49
	49	49	49	47	47	47	47	45
	41	41	40	37	35	34	29	29
	27	26	14	10				

Table 4B Crest heights in decreasing order of magnitude -1 -1 -3 -4 -9 -10 -19 -10 -11 -15 -15 -16 -16 -19 -27 -27 -29 -22 -25 -46

Table 5B	Wave periods in decreasing order of magnitude												
75.40	59.86	39.97	36.96	31.35	26.64	23.20	15.91						
15.90	15.63	14.97	14.20	14.20	13.97	13.73	13.69						
13.48	13.29	13.29	13.24	13.18	13.14	13.13	13.01						
12.86	12.77	12.68	12.66	12.63	12.52	12.52	12.43						
12.20	12.20	12.08	12.08	12.05	12.03	11.99	11.96						
11.96	11.70	11.64	11.63	11.45	11.36	11.25	11.12						
11.09	10.98	10.96	10.93	10.87	10.80	10.79	10.78						
10.72	10.61	10.41	10.39	10.30	10.26	10.02	8.88						
7.82	7.79	6.28	5.54										

Table 6B Normalised amplitudes and amplitude density

Normalised Amplitude	Fractured Waves	Density	
.02	.029	.147	
.24	.059	.294	
.4 ~ .6	.162	.809	
.68	.265	1.324	
.8 - 1.0	.265	1.324	
1.0 - 1.2	.118	.588	
1.2 - 1.4	.044	.221	
1.4 - 1.6	.044	.221	
1.6 - 1.8	.015	.074	
1.8 - 2.0	.000	.000	
2.0 - 2.2	.000	.000	
2.2 - 2.4	.000	.000	
2.4 - 2.6	.000	.000	
2.6 - 2.8	.000	.000	
2.8 - 3.0	.000	.000	
3.0 - 3.2	.000	.000	
3.2 - 3.4	.000	.000	
3.4 - 3.6	.000	.000	
3.6 - 3.8	.000	.000	
3.8 - 4.0	.000	.000	

Table 7B Normalised periods and period density

Normalised Periods	Fractured Waves	Density	
	waves		
.02	.000	.000	
.2 ~ .4	.029	.147	
.46	.191	.956	
.6 ~ .8	.618	3.088	
.8 - 1.0	.059	.294	
1.0 ~ 1.2	.000	.000	
1.2 - 1.4	.015	.074	
1.4 ~ 1.6	.015	.074	
1.6 ~ 1.8	.015	.074	
1.8 - 2.0	.000	.000	
2.0 - 2.2	.015	.074	
2.2 - 2.4	.015	.074	
2.4 - 2.6	.000	.000	
2.6 - 2.8	.000	.000	
2.8 - 3.0	.000	.000	

Table 8B Joint period and amplitude scatter diagram

Relati	ve An	plitu	de												
3.9	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
3.7	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
3.5	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
3.3	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
3.1	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
2.9	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
2.7	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
2.5	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
2.3	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	۵	.0
2.1	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
1.9	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
1.7	.0	.0	.0	1.5	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
1.5	.0	.0	.0	2.9	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
1.3	.0	.0	.0	2.9	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
1.1	.0	.0	1.5	10.3	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
.9	.0	.0	4.4	16.2	.0	.0	1.5	1.5	.0	.0	1.5	1.5	.0	.0	.0
.7	.0	.0	5.9	14.7	4.4	.0	.0	.0	1.5	.0	.0	.0	.0	.0	.0
.5	.0	.0	2.9	11.8	1.5	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
.3	.0	1.5	2.9	1.5	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
.1	.0	1.5	1.5	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
	.1	.3	.5	.7	.9	1.1	1.3	1.5	1.7	1.9	2.1	2.3	2.5	2.7	2.9

Table 9B Smoothed joint period and amplitude scatter diagram

Relati	ve An	nplitu	de												
3.9	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
3.7	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
3.5	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	9,	.0	.0	.0
3.3	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
3.1	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
2.9	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
2.7	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
2.5	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
2.3	.0	.υ	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
2.1	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
1.9	.0	.0	.1	.2	.1	0.	.0	.0	.0	.0	.0	.0	.0	.0	.0
1.7	.0	.0	.4	.7	.4	.0	.0	.0	.0	.0	.0	.0	.o	.0	.0
1.5	.0	.0	.6	1.3	.6	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
1.3	.0	.1	1.4	2.5	1.2	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
1.1	0.	.5	3.4	5.4	2.5	.1	.3	.3	.1	.1	.3	.3	.1	.0	.0
.9	.0	1.0	5.6	8.5	4.1	.5	.6	.6	.4	.3	.6	.6	.2	.0	.0
.7	.0	1.2	6.0	9.0	4.9	.7	.3	.5	.5	.3	.3	.3	.1	.0	.0
.5	.1	1.1	4.4	6.3	3.4	.5	.0	.1	.2	.1	.0	.0	.0	.0	.0
.3	.3	1.2	2.5	2.6	1.1	.1	.0	.0	.0	.0	.0	.0	.0	.0	.0
.1	.3	.9	1.1	.6	.1	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
	.1	.3	.5	.7	.9	1.1	1.3	1.5	1.7	1.9	2.1	2.3	2.5	2.7	2.9

Table 10B Amplitude and period densities

R & T	p(R)	p(T)
.0	.0000	.0000
.1	.1226	.0435
.2	.2642	.1188
.3	.4149	.2412
.4	.5614	.4218
.5	.6890	.6482
.6	.7857	.8640
.7	.8436	.9882
.8	.8606	.9818
.9	.8394	.8770
1.0	.7868	.7334
1.1	.7115	.5933
1.2	.6227	.4745
1.3	.5287	.3800
1.4	.4365	.3067
1.5	.3508	.2502
1.6	.2749	.2064
1.7	.2101	.1723
1.8	.1569	.1455
1.9	.1144	.1240
2.0	.0815	.1068
2.1	.0568	.0927
2.2	.0387	.0811
2.3	.0258	.0715
2.4	.0168	.0635
2.5	.0107	.0566
2.6	.0067	.0508
2.7	.0041	.0459
2.8	.0025	.0416
2.9	.0014	.0378
3.0	.0008	.0346
3.1	.0005	.0317
3.2	.0003	.0292
3.3	.0001	.0269
3.4	.0001	.0249
3.5	.0000	.0231
3.6	.0000	.0215
3.7	.0000	.0201
3.8	.0000	.0188
3.9	.0000	.0176

Table 11B Joint distribution of smoothed wave periods and amplitudes, p(R,T)

elati	ve Ar	nplitu	de												
3.9	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
3.7	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
3.5	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
3.3	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
3.1	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
2.9	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
2.7	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
2.5	.0	.0	.0	.0	.1	.1	.0	.0	.0	.0	.0	.0	.0	.0	.0
2.3	.0	.0	.0	.1	.2	.1	.1	.0	.0	.0	.0	.0	.0	.0	.0
2.1	.0	.0	.0	.2	.4	.3	.1	.1	.0	.0	.0	.0	.0	.0	.0
1.9	.0	.0	.0	.4	.7	.5	.3	.2	.1	.0	.0	.0	.0	.0	.0
1.7	.0	.0	.1	.8	1.2	.9	.5	.3	.2	.1	.1	.0	.0	.0	.0
1.5	.0	.0	.3	1.5	1.8	1.3	.8	.5	.3	.2	.1	.1	. 1	.0	.0
1.3	.0	.0	.6	2.3	2.4	1.7	1.1	.7	.4	.3	.2	.2	.1	.1	.1
1.1	.0	.0	1.3	3.1	2.8	1.9	1.3	.8	.6	.4	.3	.2	.2	.1	.1
.9	.0	.2	2.2	3.6	2.9	1.9	1.3	.9	.6	.5	.4	.3	.2	.2	.2
.7	.0	.6	3.0	3.4	2.4	1.6	1.1	.8	.6	.5	.4	.3	.2	.2	.2
.5	.0	1.4	3.1	2.4	1.6	1.1	.7	.5	.4	.3	.3	.2	.2	.1	.1
.3	.3	2.0	1.9	1.1	.7	.5	.3	.2	.2	.1	.1	.1	.1	.1	.1
.1	.7	.8	.4	.2	.1	.1	.1	.0	.0	.0	.0	.0	.0	.0	.0
	.1	.3	.5	7	.9	1.1	1.3	1.5	1.7	1.9	2.1	2.3	2.5	2.7	2.9

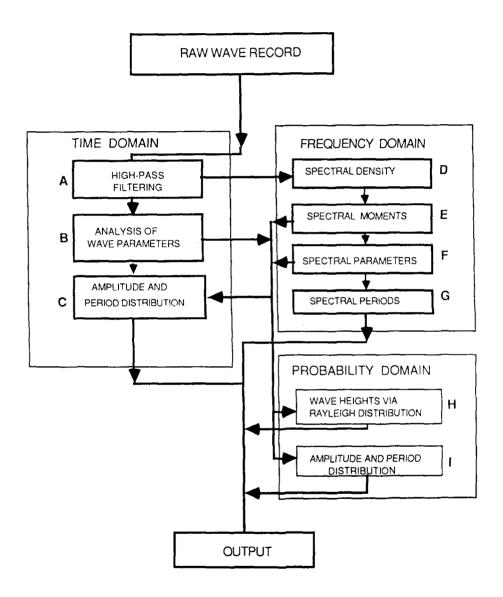


Figure 1 Wave analysis flow chart.

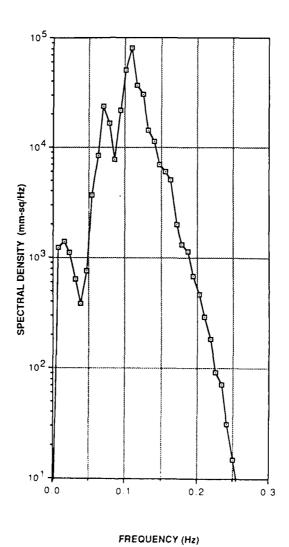
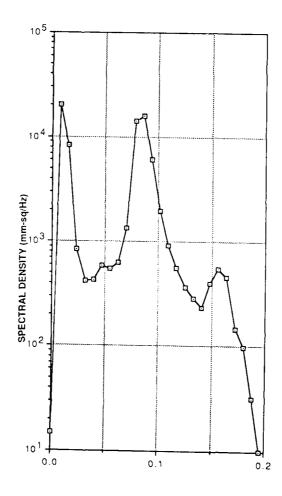


Figure 2 Spectral density for wave record A.



FREQUENCY (Hz)

Figure 3 Spectral density for wave record B.

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This note outlines a computer program to replace the manual Draper-Tucker analysis of bottom pressure fluctuations induced by surface waves.